

Derivatives

Chain Rule

1. Find the derivative of $f(x) = \cos^3\left(\frac{1}{3x+1}\right)$.
2. Find the derivative of $f(x) = x^5 \cos(3x)$.
3. Find the derivative of $\sqrt{\sin(2x)}$.
4. Find the derivative of $\sin(\sqrt{x})$.
5. Find the derivative of $\cot(3x^2)$.

Inverse Functions

6. Find the derivative of $\operatorname{arccot}(x)$.
7. Find the derivative of $\arcsin(x)$.
8. Let $f(x) = xe^x$ and let g be the inverse function of f . Given that $f(1) = e$, find $g'(e)$.
9. Let $f(x) = x^5 + 4x^3$ and let g be the inverse function of f . Given that $f(1) = 5$, find $g'(5)$.
10. Let $f(x) = x^5 + x + 1$ and let g be the inverse function of f . Find the derivative of g at $(3, 1)$.

Implicit Differentiation

11. Find the derivative of the function $y^2(6 - x) = x^3$ at $(3, 3)$.
12. Find $\frac{dy}{dx}$ given that $\frac{1}{y} + \frac{1}{x^2} = 1$.
13. Let $y^2 = x^2(x - 1)$. At what points is $\frac{dy}{dx}$ not defined?
14. Find $\frac{dy}{dx}$ if $\ln(xy) = e^y$.
15. Find when the curve $x^4 = 2x^2 - y^2$ has a horizontal derivative.

L'Hopital's Rule

16. Find $\lim_{x \rightarrow \infty} \sqrt{2x+1} - \sqrt{x+1}$.
17. Find $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2-1}}$.
18. Find $\lim_{x \rightarrow \infty} x^{1/x}$.
19. Find $\lim_{x \rightarrow 0^+} x^x$.
20. Find $\lim_{x \rightarrow 0} \frac{\cos(x) - 1 + \sin(x)^2/2}{x^4}$.

Application

Optimization

21. Find the area of the smallest triangle formed by the x axis, y axis, and a line that goes through the point $(1, 1)$.
22. Find the largest rectangle that can be inscribed into a semicircle of radius 2 so that one side of the rectangle is part of the diameter of the semicircle.
23. Find the point on the curve $y = 1 - \sqrt{x}$ closest to $(1, 1)$.
24. A rectangle is inscribed under the curve $\sin x$ for $0 \leq x \leq \pi$. This rectangle has two vertices on the curve and one side on the x axis. What is the maximum possible area of such a rectangle.
25. What is the point on $y = e^x$ closest to $(1, 0)$?

Related Rates

26. A ball of light is bobbing up and down and whose position is given at a time t by $4 + \sin(2t)$. A man who is $2m$ tall is standing $10m$ away. How fast is the length of his shadow changing when $t = 0$?
27. A conical cup that is $6cm$ wide at the top and $5cm$ tall is filled with water is punctured at the bottom and water is coming out at a rate of $10^{-6}\pi m^3/s$. Initially, the cup is filled. How fast is the height of the water changing when the height is $3cm$?
28. Sand is being dumped in a conical pile whose radius and height always remain the same. If the sand is being dumped in at a rate of $2\pi m^3/hr$, how fast is the height of the sand changing when the pile is $5cm$ tall?

29. A kite is flying at a current altitude of $100m$. The kite slowly flies further and further away as the string length increases at a rate of $2cm/s$. Assuming the altitude does not change, how fast horizontally is the kite moving when the angle the string forms with the ground is $\pi/4$?
30. A ladder $13m$ tall is lying against a wall. The bottom of the ladder is pulled out at a rate of $10cm/s$. How fast is the area of the triangle formed by the ladder, wall, and floor changing when the bottom of the ladder is $5m$ away from the wall?

Taylor Series

31. Use the third order approximation to find $\sin(0.5)$.
32. Approximate $\sqrt{99}$ using a quadratic regression.
33. Use the second order approximation to find $\ln 1.01$.
34. Use the second order approximation to $\sqrt[3]{8.1}$.
35. Use the quintic order approximation to find e .

Newton's Method

36. Use Newton's method once to approximate $\sqrt[3]{8.1}$.
37. Approximate $\sqrt{99}$ using Newton's method once.
38. Find the critical points of $g(x) = \sin(x) - x^2$
39. Find the unique solution to $(\pi - 2x) \cos(x) = 2 \sin(x)$ on the interval $[0, 1]$ using Newton's method with an initial guess of $x = \frac{\pi}{4}$.
40. Find when $\cos(x) = x$ using Newton's method and an initial guess of $x = \frac{\pi}{6}$.

Functions

Domain/Range

41. Find the domain of $y = \sqrt{9 - (2x + 3)^2}$.
42. Find the domain of $y = \frac{1}{\sqrt{3-x}}$.
43. Find the domain and range of $2 - \arccos(3x + 2)$.
44. Find the domain of $\frac{\ln(x+3)}{\sqrt{2-x}}$.
45. Find the domain of $\sqrt{\frac{3-x}{1-x}}$.

Inverse Functions

46. Find the inverse of $f(x) = \frac{-2}{x} - 1$.
47. Find the inverse of $\frac{4+\sqrt{3x}}{5}$.
48. Find the inverse to x^2 on $(-\infty, 0]$.
49. Find the inverse to e^{2x+3} .
50. Find the inverse to $-\sqrt{\ln x}$.

Graphing

51. Sketch the graph of $f(x) = \frac{x}{x^2 + 1}$.
52. Sketch the graph of $f(x) = x + \frac{1}{x-1}$.
53. Sketch the graph of $f(x) = 3 - 15x - 6x^2 + x^3$.
54. Sketch the graph of $f(x) = \frac{x-1}{x+1}$.
55. Sketch the graph of $f(x) = e^x + 2e^{-x}$.

1 True/False

56. True False You can tell what the domain and range of an inverse function is only from the domain and range of the original function
57. True False The horizontal line test tells us whether a function is injective or surjective.
58. True False The range of e^x is $[0, \infty)$.
59. True False If $\lim_{x \rightarrow 0} f(x)$ exists, then $\lim_{x \rightarrow 0} f(x) = f(0)$.
60. True False Extrema of a function must occur when the derivative is 0, when it doesn't exist, or at the endpoints.
61. True False The expression 0^∞ is an indeterminate.
62. True False The continuity law for subtraction follows from the limit law for subtraction.
63. True False The continuity of a constant function follows from limit laws.

64. True False The continuity law for rational functions follows only from the limit laws for ratios.
65. True False If f is not continuous at $x = c$, then f is not differentiable at $x = c$.
66. True False We can use the power rule to find the derivative of x^x .